Correlation Length and Force Phasing of a Rigid Cylinder Subject to VIV

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Introduction : The fluctuations of the lift forces acting on a free rigid cylinder subject to vortex-induced vibrations (VIV) depend on the degree of three dimensionality present in the near wake of the body. The spanwise correlation length of the flow gives an accurate measure of this three dimensionality and consequently gives some indications of the magnitude of the cylinder response amplitude. However, this information is not enough by itself as it does not provide a measure of the phase difference between structure motion and lift forces for instance. In studies of VIV the value of spanwise correlation length is very important as many empirical models rely on it [1]. It is also important in numerical simulation studies as it provides guidance for the choice of spanwise numerical resolution; mesh refinements along the spanwise direction are often overlooked as the strongest flow gradients occur along the streamwise and crossflow directions. However, related studies and experimental measurements of correlation length are relatively few in number [2, 3]. For stationary cylinders, experimental measurements of correlation length based on the autocorrelation function were obtained only recently [4, 5]. For moving, rigid, free cylinders, detailed measurements of cross-correlation between lift forces measured at the two ends of the cylinder can be found in [6]. The influence of nominal reduced velocity on the correlation length and phasing of the near wake flow remains to be explored.

Formulation : Here, we present direct numerical simulations (DNS) results of vortex-induced vibrations of a smooth rigid cylinder with aspect ratio $L/d = 26$ and mass ratio $m = 2$ using spectral/hp elements method [7]. A boundary-fitted coordinate system approach is employed and an implicit second-order Newmark integration scheme is used to solve for the structure. The Reynolds number is in the subcritical range resulting in a turbulent near-wake. We set the structural damping to be zero as we are interested in the maximal response of the system. We also only consider the dominant motion in the cross-flow direction and also preclude any motion of the structure in the streamwise direction. We test three different reduced velocities for the natural frequency of the oscillator. We choose our reference reduced velocity $V_r = V_{ref}$ to be based on the Strouhal frequency of the two-dimensional stationary cylinder wake. Past three dimensional simulations using this value have provided maximal structural response. We choose the other two reduced velocities to be above and below the reference value by $\pm 10\%$.
We define the autocorrelation function as follows:

$$R_{uu}(l; x, y) = \frac{\overline{u(x, y, z, t)u(x, y, z - l, t)}}{\overline{u^2(x, y, z, t)}}, \quad (1)$$

where the bar denotes averaging over time and over z-planes. Also, $u(x, y, t)$ is the fluctuation obtained after we subtract the mean quantity, i.e. averaged in time and span (z) at the (x, y) point.

To quantify the phase difference we employ *complex demodulation analysis*, which is a more general approach than harmonic analysis in dealing with non-exact periodic time series (see Bloomfield (1976) [8]). A complex demodulation of a time series $S(t_n)$ with a dominant frequency component λ will give a time varying amplitude $R(t)$ and phase $\Phi(t)$ such that:

$$S(t_n) \approx R(t_n)e^{i(\lambda t_n + \Phi(t_n))} \quad (2)$$

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The results of this analysis and in particular, the effect of the reduced velocity change on the phasing between cross-flow displacement and lift force, are under investigation and will be presented at the conference.

Results : In Figure 1, we plot the absolute value of the streamwise and cross-flow velocity autocorrelation functions (spanwise velocity is not presented here) for two points in the near-wake for three different structural reduced velocities.

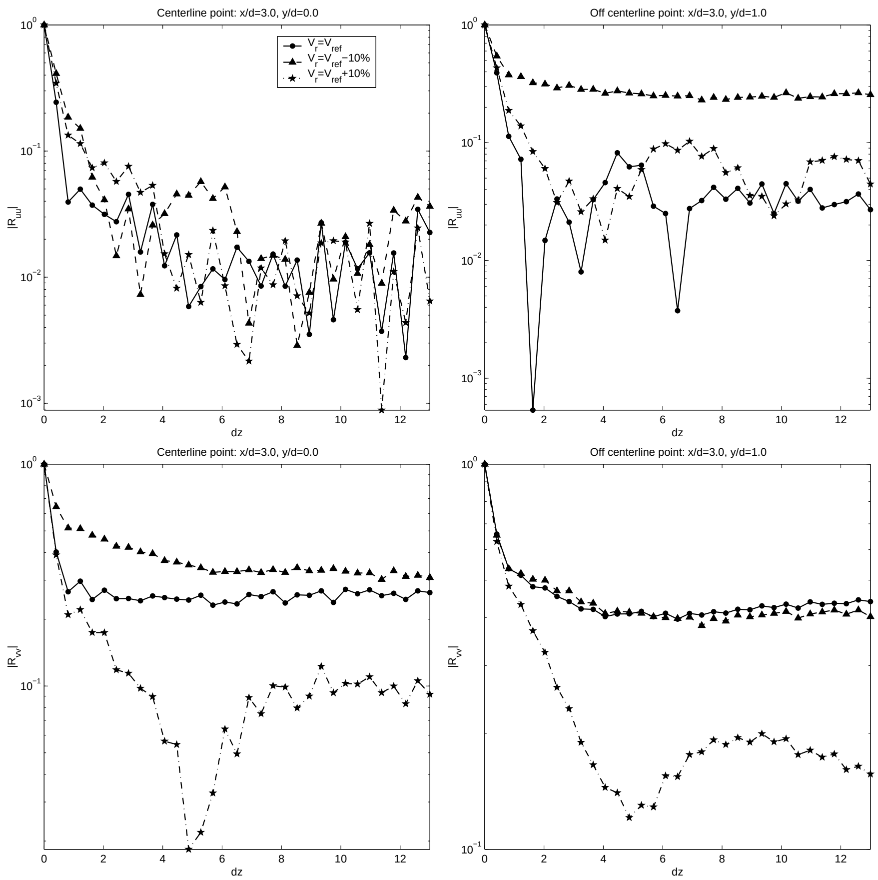

Figure 1: Streamwise and cross-flow velocity autocorrelation functions in the free rigid cylinder near-wake at a centerline point ($x/d = 3; y/d = 0$) (left) and an off-centerline point ($x/d = 3; y/d = 1$) (right), for three different reduced velocities.

The autocorrelation function $|R_{vv}|$ (see figure 1, second row) indicates a high degree of correlation for the rigidly oscillating cylinder with the reference reduced velocity close to the Strouhal frequency. Also, a decrease (-10%) in the reduced velocity increases slightly the correlation in the wake but decreases significantly the maximum amplitude response of the structure (not represented here). However, when the reduced velocity is larger ($+10\%$) and approaches a value of 5, a sharp drop in the correlation occurs. The near wake becomes less organized but the maximum amplitude response of the structure is not significantly affected by this change. Finally, the values for $|R_{ww}|$ (not presented) are comparatively much smaller for all cases, especially in the case of the centerline point. These results are in good agreement with the experimental results by Hover and al. ([6] and private communication).

References

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