

Bifurcations at 2:1 Resonance in the Mathieu Oscillator With Time Delay

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Resonance phenomenon in linear and nonlinear oscillators is of importance in several areas of engineering mechanics, and the distinguishing feature of such oscillators is that the imposed displacement or excitation appear usually in the governing equations of motion in the form of a coefficient or parameter. In some applications, where component materials are exposed to randomly microstructural grain and porosity sizes, the parameter may consist of a harmonic term on which a stochastic fluctuation is superposed. It is well known that regions of instability for a Mathieu oscillator are found when the ratio of the frequency of the harmonic excitation to twice the natural frequency of the oscillator is in the neighbourhood of the values $1/p$, where p is an integer. The most important of these corresponds to $p = 1$ and is known as the region of subharmonic resonance. In this paper, a method of obtaining a sufficient almost-sure stability bifurcations for the Mathieu oscillator with a parametric Wiener process and restoring time delay force is presented. First, we compute the conditions of Hopf bifurcation to determine critical parameter values at which the possible bifurcations may occur, and then using a reduction approach, we show that the bifurcations have a Markovian centre subspace with dimension 2 for varying time delay. Series of transformations are applied to convert the Wiener process in the Markovian centre subspace into Gaussian white noise of amplitude and phase equations. Subharmonic resonance regimes and bifurcations displayed qualitatively. The results of this investigation are of importance in the study of stability of thread machining on a cylindrical shaft. During this type of machining, persistent periodic motions causing instability, can be experienced at every tool cut.