A general algorithm for coupling Lagrangian-Shell with Eulerian-Fluid Formulations

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Abstract. We propose a computational method for the coupled simulation of the large dynamic deformations of shell structures interacting with a compressible fluid flow. This work is an extension to shells of the general Eulerian-Lagrangian coupling strategy for bulk solids presented previously (Cummings et al., 2002; Meiron et al., 2001). An Eulerian finite volume formulation is adopted for the fluid whereas a Lagrangian formulation based on subdivision finite elements is adopted for the shell response. The coupling between the Eulerian fluid solver and the Lagrangian shell solver is accomplished *via* a novel technique based on level sets.

The Eulerian fluid algorithm employed (Samtaney and Zabusky, 1994; Samtaney and Meiron, 1997) solves the equations of inviscid compressible flow written in strong conservation form. The spatial discretization employed corresponds to a finite volume formulation on Cartesian grids. The spatially-discretized equations are integrated in time by recourse to the second-order Runge-Kutta algorithm. In the calculations presented we employ the ideal-gas equation of state and the Godunov scheme to compute the fluxes at cell interfaces. More details about the computational fluid dynamics approach are given in the cited references.

The mechanical shell response is computed with the recently introduced subdivision finite elements (Cirak and Ortiz, 2001; Cirak et al., 2000). The Kirchhoff-Love kinematic assumption for shells is adopted. This allows for arbitrary large displacements and rotations of the shell and includes membrane stretching effects. As it is well known, the Kirchhoff-Love energy functional of the thin-shell depends on the first and second order derivatives of displacements. It is known from approximation theory that the convergence of the related finite-element procedure requires smooth C^1-continuous shape functions. On unstructured meshes it is not possible to ensure strict slope continuity across finite elements when the elements are endowed with purely local polynomial shape functions and the nodal degrees of freedom consist of displacements and slopes only. Inclusion of higher derivatives among the nodal variables lead to several well known difficulties, e.g. spurious oscillations in the solution, nonphysical higher order derivatives at the boundary vertices, or complex schemes for nonsingular parameterization of the derivatives for large rotations. These difficulties are avoided by the judicious choice of *"nonlocal"* subdivision shape functions for the discretization of Kirchhoff-Love type shell theories on unstructured meshes. The displacement field within one finite element is interpolated through the displacements of the vertices attached to the element and the immediately adjacent vertices in the mesh. In the resulting shell discretization, the nodal displacements of the subdivision finite elements constitute the only unknowns of the shell problem.

The coupling algorithm consists of exchanging boundary information between the fluid and the solid solvers and using this information to enforce the conservation laws at the boundary in each solver as follows. The reflective (zero mass-flux) boundary conditions in the fluid is modified to account for the motion of the boundary and enforced through extrapolation to ghost cells outside the fluid domain. A key ingredient in the proposed strategy is an efficient algorithm for the computation of the signed distance function to the fluid-solid boundary (Mauch, 2001). The signed distance function provides an implicit description of the geometric location of the fluid-solid boundary as its zero level set. This information is employed for locating the boundary between the fluid and the solid and, more importantly, the narrow band of ghost cells next to the boundary where the velocity and pressure fields need to be extrapolated to enforce the zero mass flux boundary condition. The transfer of momentum from the fluid to the shells is accomplished *via* a consistent integration of the fluid pressure as traction boundary conditions on the shell finite element mesh. The fluid pressure on the solid boundary is obtained by interpolation from the fluid grid reusing the level set information. The exchange of boundary information between the two solvers takes place at the beginning of the time step.

The resulting algorithms are implemented on the Virtual Test Facility(Cummings et al., 2002). Details of its architecture as well as of its implementation and scalability properties in large numbers of processors may be found in the cited reference. The proposed approach is robust and highly scalable. The proposed method avoids the robustness issues associated with cut-cell approaches–specially in three dimensions– and furnishes, in effect, a general means of coupling the interaction of high-speed flows with highly-deformable solids.

As a means of demonstrating the feasibility and power of the method to model complex interactions of shocks propagating in a gas bounded by a highly-deformable shell structure, a simulation of the deployment of an airbag is presented. The simulation corresponds to the case of an initially-flat airbag made of an elastic fabric whose Young's Modulus and Poisson's ratio are $E = 10^7 Pa$ and $\nu = 0.3$, respectively. The initial diameter of the airbag in its flat initial configuration is $D = 0.5m$. The properties of the gas are: $\rho = 1.03 \frac{Kg}{m^3}$ and $\gamma = 1.4$. The gas enters the airbag with a pressure $p = 10^8 Pa$ generating a shock that propagates inside the airbag and interacts with the airbag walls. The pressure of the gas at the inlet is maintained throughout the simulation. Figures 1-3 show a sequence of snapshots of the simulation. The deformed airbag meshes are shown on the left of each figure and the density isocontours on the centerplane are shown on the right of each figure. The portions of the fluid grid that are external to the airbag have been left out for visual clarity by making use of the level set function one more time at postprocessing. Important features of the mechanics of the airbag deployment process can be observed in these figures, including the high-frequency wrinkling modes of the airbag fabric and the shock reflections of the gas on the deforming airbag walls. The ability of the method to capture these complex features of the coupled interaction between the flow and the airbag fabric is particularly noteworthy.

Keywords: fluid-shell interaction, level sets, Eulerian-Lagrangian approaches

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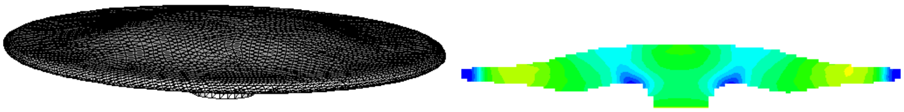

Figure 1. Simulation of airbag deployment: deformed finite element shell mesh and isocontours of gas density in fluid X-Z centerplane step 1000

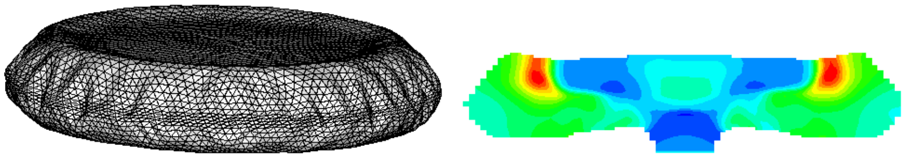

Figure 2. Simulation of airbag deployment: deformed finite element shell mesh and isocontours of gas density in fluid X-Z centerplane step 2000

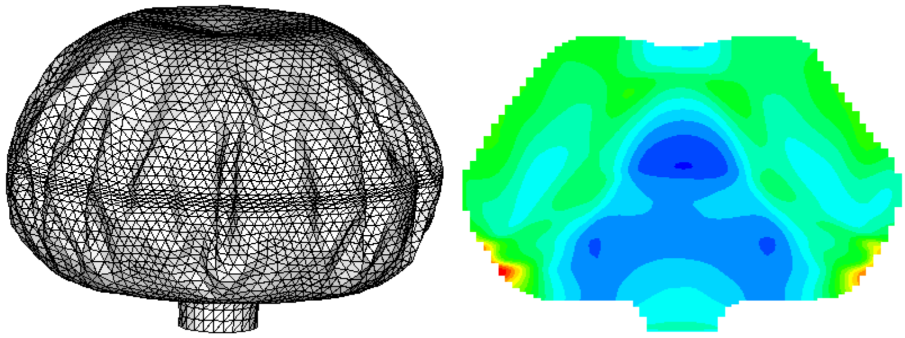

Figure 3. Simulation of airbag deployment: deformed finite element shell mesh and isocontours of gas density in fluid X-Z centerplane step 3000