Forced Dynamic Uplift of Floating Plates

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Abstract

When a floating plate is subjected to forced subsurface uplift, the dynamics of the plate and fluid are intertwined in a fully-coupled manner. A fully nonlinear analysis requires large-displacement theory for the plate and fully viscous theory for the fluid. Because of the fully coupled nature of the problem, and the inherent level of nonlinearity, an analytical formulation is followed by numerical computation. A significant component of the latter involves code validation. That is where the present study comes in. In this paper, a linearized mathematical model of the problem is analyzed. The plate is taken to be thin and linearly elastic, and the fluid is taken to be nonviscous and irrotational. Though only weakly coupled, the linearized model identifies the nondimensional parameters of the problem and provides useful information for both experimentalists and numerical simulators alike. The linearized model is used to study two canonical floating plate problems.

In the linearized model, a characteristic length scale that emerges is

$$\ell = \left(\frac{D}{\rho_f g}\right)^{1/4},$$

where D is the flexural rigidity of the plate, ρ_f is the density of the fluid, and g is the acceleration due to gravity. The nondimensional parameters of the problem are H and μ, where $H\ell$ is the depth of the fluid and

$$\mu = \frac{\rho_p h}{\rho_f \ell},$$

where ρ_p is the density of the plate. A schematic of a floating plate with a concentrated point load at the origin is shown in Figure 1. Axisymmetric coordinates r, z are centered at the midsurface of the

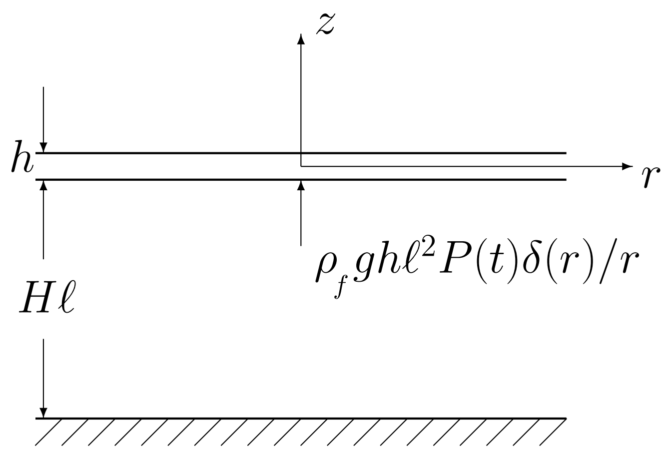

Figure 1: An infinite plate of thickness h, floating on a fluid of depth $H\ell \gg h$, is subjected to a concentrated uplift force $\rho_f gh\ell^2 P(t)\delta(r)/r$.

plate. If the uplift displacement w is specified, as it often is, the uplift force P is the solution of a Volterra integral equation of the first kind and of convolution type;

$$w(\tau) = \int_0^\tau k(\tau - u)\varphi(u)\, du$$

where $\tau = t\sqrt{g/\ell}$ is nonndimensional time and $\varphi(\tau) = P'(\tau)$. Such problems are inherently ill-posed.

It turns out that many of the practical situations surveyed reduce to one of two canonical problems. In one, the upward velocity is constant, and in the other, the upward acceleration is constant. This paper will discuss these two canonical problems and the problems encountered in determining P to a known level of accuracy. Accurate numerical results are useful for benchmark purposes in a fully coupled nonlinear simulation. Accurate numerical results are also useful in another way. Designers and experimentalists need to know how P depends on the parameters μ and H. This parameter dependence is not immediately evinced by the integral equation. The numerical results are used in this paper to infer what this dependence is.