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Frequency response and the existence of a critical mass for an elastically-mounted cylinder

R. Govardhan[1] & C.H.K. Williamson[2]

[1] Department of Mechanical Engineering,
Indian Institute of Science, Bangalore - 560012, INDIA
raghu@mecheng.iisc.ernet.in

[2] Sibley School of Mechanical & Aerospace Engineering,
Upson Hall, Cornell University, Ithaca, NY 14853, USA
cw26@cornell.edu

EXTENDED ABSTRACT

In the present work, we study the transverse vortex-induced vibrations of an elastically-mounted rigid cylinder in a fluid flow at low mass-damping values. The response in this case consists of three distinct branches; namely the initial, upper and lower. The distinct vortex formation modes associated with each of these three branches are discussed in detail in Govardhan and Williamson (2000).

The oscillation frequency (f) of an elastically-mounted cylinder depends primarily on the mass ratio (m*), as discussed for example in Bearman (1984). This may be seen clearly from the equation for the cylinder frequency response ($f^* = f/f_N$) shown below, which is formulated here along the lines of Khalak and Williamson (1999), as follows:

$$f^* = \sqrt{\frac{(m^* + C_A)}{(m^* + C_{EA})}} \qquad (1)$$

where C_{EA} is the effective added mass due to wake vortex dynamics, and C_A is the potential added mass (C_A=1.0 for a circular cylinder).

At high mass ratios, the large m* values overwhelm the effect of the effective added mass (C_{EA}), thus yielding $f^*=f/f_N \approx 1.0$. Hence, at high mass ratios, as in Feng (1968), the response frequency (f) is close to the natural frequency (f_N) in the synchronization regime. On the other hand, at low m* the effective added mass (C_{EA}) influences f* significantly, and results in a marked deviation of f* from unity. This has been seen for example in more recent experiments of Khalak & Williamson (1999) and Govardhan & Williamson (2000).

An example of an amplitude and frequency response at low mass ratio is shown in figure 1. A striking aspect of this is the almost constant value for the lower branch frequency over the complete response branch. This constant level of frequency is observed at every mass ratio investigated (m*=0.8 to 20) and seems to be a general characteristic of this lower branch. This observation is also supported by the frequency data of Hover et al. (1998) and Khalak & Williamson (1999), both at similar low mass-damping values.

A large set of data for the Lower branch frequency (f^*_{LOWER}) plotted versus m*, is shown in Figure 2. This data is from our own experiments, and from Hover et al. (1998), Khalak & Williamson (1999), and Anand (1985). The data collapse very well onto a single curve. Further, it can be shown that a single value of the effective added mass (C_{EA}) represents the complete set of lower branch frequency data. This single value of C_{EA} may be found as the best fit of C_{EA} in equation (1) which represents the experimental data of Figure 4. From this analysis, we find C_{EA} = -0.54, and we thereby deduce the following Lower-branch frequency equation as:

$$f^*_{LOWER} = \sqrt{\frac{(m^* + 1.0)}{(m^* - 0.54)}} \qquad (2)$$

This curve is drawn through the data in Figure 4, and it represents the data very well. The expression for f^*_{LOWER} in equation (2) provides a practical and simple means to calculate the highest frequency in the synchronization regime, if we are provided with the mass ratio, m*.

An important consequence of equation (2) is that the frequency becomes infinite as the mass ratio reduces to the limiting value of 0.54. Therefore we conclude that there exists a critical mass ratio:

$$\textit{Critical mass ratio}, \quad m^*_{CRIT} = 0.54 \qquad (3)$$

The existence of a critical mass is, for us, a surprising and interesting result.

At mass ratios below the critical mass ratio, $m^* < m^*_{CRIT}$, it can be shown that the lower branch ceases to exist, and the upper branch continues indefinitely. Hence for $m^* < m^*_{CRIT}$, the regime of large-amplitude resonant response extends to infinite normalized flow speed !

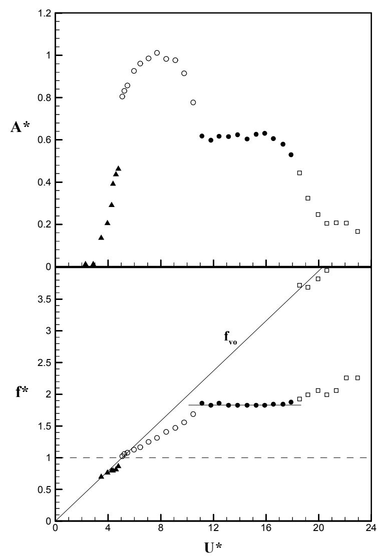

Figure 1. Amplitude and frequency as a function of normalized velocity $U^*=U/f_ND$, for very low mass ratio, $m^*=1.19$. ▲, initial; o, upper; •, lower; □, desynchronized.

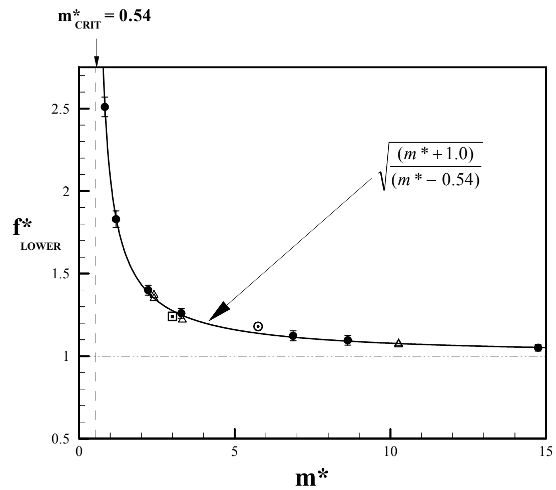

Figure 2. Variation of the lower-branch frequency (f^*_{LOWER}) as a function of the mass ratio m*. The equation for f^*_{LOWER} fits the data remarkably well, and indicates a dramatic increase in f^*_{LOWER} as we approach the critical mass ratio, $m^*_{CRIT} = 0.54$. •, Present data; Δ, Khalak & Williamson (1999); ⊡, Hover et al. (1998); ⦿, Anand (1985).

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